GRAVITATIONAL WAVES FROM COSMIC STRINGS

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Gravitational waves from cosmic strings are generated in the first fractions of a second after the Big Bang, potentially providing a unprecedented probe of the early universe. We discuss the key dynamical processes underlying calculations of the stochastic background produced by a string network and we detail the parameter dependencies of the resulting spectral density $\Omega_{\rm gr}(f)$. The present constraints on the cosmic string mass scale μ arising from the millisecond pulsar timings and primordial nucleosynthesis are discussed, justifying our present conservative bound $G\mu/c^2 < 5.4(\pm 1.1)\times 10^{-6}$. We then discuss the strong prospect of detecting (or ruling out) cosmic string background with the next generation of gravitational wave experiments. Comparison is also made with alternative cosmological sources of gravitational waves such as inflation and hybrid topological defects.

1 Introduction

An outstanding achievement of modern cosmology has been the detection of anisotropies in the cosmic microwave background[1]. These anisotropies provide a snapshot of the universe about 400,000 years after the Hot Big Bang, just as the universe became transparent to electromagnetic radiation. Despite this observational triumph, as yet it has been insufficient to differentiate between competing paradigms for galaxy formation, nor has it shed much light on cosmological processes taking place before photon decoupling. There are, however, other types of radiation. Gravitational radiation, as considered in the present work, may penetrate through this electromagnetic surface of last scattering, and travel virtually unaffected since emission. Of course, for gravitons this remarkable transparency is due to their very weak interaction with ordinary matter which, in turn, makes them difficult to observe. However, pioneering experiments have been proposed which could detect a stochastic

background of gravitational waves generated in the early universe over a range of frequencies.

Cosmic strings are line-like topological defects which may have formed during a phase transition in the early universe[2,3]. Strings which formed with a mass-per-unit-length μ such that $G\mu/c^2 \sim 10^{-6}$ may be responsible for the formation of the large-scale structure and cosmic microwave background anisotropy observed in the universe today. In general, a network of strings will evolve towards a self-similar scaling regime by the production of loops and subsequent emission of gravitational radiation. Since the sources of these gravitational waves come from many horizon volumes this background will be stochastic and is likely to appear as noise on a gravitational wave detector.

A stochastic background of gravitational waves is normally quantified by the relative spectral density $\Omega_{\rm g}(f)$ given at a frequency f. That is, the energy density in gravitational radiation in an octave frequency bin centred on f, relative to the critical density of the universe. This is directly related to the dimensionless wave amplitude ($h_c \propto \sqrt{\Omega_g}/f$) which is measured experimentally.

In the case of gravitational radiation produced by a cosmic string network, for a given frequency today we may identify a characteristic time at which the waves were emitted. Assuming that the radiation is emitted at a time $t_{\rm e}$ before equal matter-radiation ($t_{\rm e} < t_{\rm eq} \sim 4,000$ years) and that it is created with a wavelength comparable to the horizon $\lambda(t_{\rm e}) \sim t_{\rm e}$, then the frequency today is given by $f \sim z_{\rm eq}^{-1}(t_{\rm eq}t_{\rm e})^{-1/2}$ where the redshift is $z_{\rm eq} \sim 2.3 \times 10^4 \Omega_0 h^2$. In particular those gravitational waves created at the electroweak phase transition will have a frequency $f \sim 10^{-3}$Hz, whereas those created at the time of radiation-matter equality will have a frequency $f \sim 10^{-13}$Hz.

The relevant constraints on stochastic backgrounds are due to the measured timing residuals in pulsar signals and also from Big Bang nucleosynthesis. The most recent analysis of pulsar signal arrival times[4] gives the limit on the spectral density of gravitational radiation

$$\Omega_{\rm gr}(f) \equiv \frac{f}{\rho_{\rm crit}}\frac{d\rho_{\rm gr}}{df}\,|_{f_{\rm obs}} < 9.3 \times 10^{-8} h^{-2} \quad (95\%\,{\rm CL}) \tag{1}$$

in a logarithmic frequency interval at $f_{\rm obs} = (8\,{\rm yrs})^{-1}$. This analysis corrects errors in previous work[5,6], and uses an improved method for testing the hypothesis that the timing noise is due to a stochastic gravitational wave background. For these reasons we use the latest bound, equation (1), to obtain a constraint on $G\mu/c^2$ for given values of the cosmic string and cosmological parameters.

The bound from nucleosynthesis is slightly different, as it constrains the entire spectrum. The standard constraint quoted is that the total energy den-

sity in gravitational waves at the time of nucleosynthesis must be less than 5.4% of the critical density, or

$$\Omega_{\rm gr} = \int \frac{d\omega}{\omega} \Omega_{\rm gr}(\omega) \approx 0.162 (N_\nu - 3)\Omega_{\rm r} < 0.054 \,, \tag{2}$$

which can be thought of as a contribution from an extra neutrino species. This constraint is somewhat weakened at present, due to uncertainties in the observed element abundances. For this reason, we shall discuss constraints from a range of values for N_ν, the effective number of neutrino species.

In this proceedings, based in part on work in ref.[7], we review the status of the stochastic background produced by cosmic strings. The details of the spectrum of gravitational radiation due to cosmic strings has been previously considered elsewhere[8]. Here we benefit from recent work[9], which suggests that the effect of the radiative back-reaction on a string loop is to damp out the higher oscillation modes. We illustrate the effects of a non-standard thermal history, low-Ω universes and discuss hybrid systems of defects, such as strings and domain walls or strings connected by monopoles. Finally, we discuss the opportunities for detection.

2 Cosmic string loop radiation spectrum

The spectrum of gravitational radiation emitted by a network of cosmic string loops is obtained using a background $\Omega = 1$ FRW cosmological model, an extended one-scale model for the evolution of a network of cosmic strings, and a model of the emission of gravitational radiation by cosmic string loops. The procedure by which the spectrum is computed has been presented in detail by Caldwell and Allen[8]. In this section we discuss the model for radiation by an individual loop.

The model of the emission of gravitational radiation by cosmic string loops is composed of the following three elements.

1. A loop radiates with power $P = \Gamma G \mu^2 c$. The dimensionless radiation efficiency, Γ, depends only on the loop configuration, rather than overall size. Recent studies of realistic loops[10] indicate that the distribution of values of the efficiency has a mean value $\langle \Gamma \rangle \approx 60$.

2. The frequency of radiation emitted by a loop of invariant length L is $f_n = 2n/L$ where $n = 1, 2, 3, ...$ labels the oscillation mode.

3. The fraction of the total power emitted in each mode of oscillation n at frequency f_n is given by the coefficient P_n where

$$P = \Big(\sum_{n=1}^{\infty} P_n\Big) G\mu^2 c = \Gamma G\mu^2 c. \tag{3}$$

   Analytic and numerical studies suggest that the radiation efficiency coefficients behave as $P_n \propto n^{-q}$ where q is the spectral index.

This model has several shortcomings. First, the spectral index q has not been well determined by the numerical simulations. Numerical work[11] suggests $q = 4/3$ as occurs with cuspy loops, loops along which points momentarily reach the velocity of light, based on simulations of a network of cosmic strings. However, these simulations have limited resolution of the important small scale features of the long strings and loops. Hence, the evidence for $q = 4/3$ is not compelling. Analytic work[12] suggests that $q = 2$, characteristic of kinky loops, loops along which the tangent vector changes discontinuously as a result of intercommutation, may be more realistic. Second, the effect of back-reaction on the motion of the string has been ignored. In this model, a loop radiates at all times with a fixed efficiency, in all modes, until the loop vanishes. As we shall next argue, the back-reaction will result in an effective high frequency cut-off in the oscillation mode number. Thus, the loop will only radiate in a finite number of modes, and hence in a finite range of frequencies. The resolution of these issues may have strong consequences for the entire spectrum produced by a network of strings.

Recent advances in the understanding of radiation back-reaction on global strings suggest various modifications to the simplified model of emission by cosmic string loops. It has been shown[13] that there are remarkable similarities between gravitational radiation and Goldstone boson radiation from strings, which we believe allow us to make strong inferences as to the nature of gravitational radiation back-reaction. For example, the same, simple model for the emission of gravitational radiation by cosmic strings may be transferred over to global strings: in the absence of Goldstone back-reaction, global string loops radiate at a constant rate, at wavelengths given by even sub-multiples of the loop length, with an efficiency as described by an equation similar to (3). Hence, our argument proceeds as follows. Fully relativistic field theory simulations of global strings have been carried out[9], where it was observed that the power in high oscillation modes is damped by the Goldstone back-reaction on periodic global strings. An analytic model of Goldstone back-reaction[9], as a modification of the classical Nambu-Goto equations of motion for string, was developed which successfully reproduces the behaviour observed in field theory

simulations. That is, high frequency modes are damped rapidly, whereas low frequency modes are not. Thus, we are motivated to rewrite equation (3) for global strings, and by analogy for cosmic strings, as

$$P = \Big(\sum_{n=1}^{n_*} P_n\Big) G\mu^2 c = \Gamma G\mu^2 \tag{4}$$

where n_* is a cut-off[14] introduced to incorporate the effects of back-reaction. By comparing the back-reaction length-scale to the loop size, we estimate that such a cut-off should be no larger than $\sim (\Gamma G\mu/c^2)^{-1}$. The ongoing investigations of global and cosmic string back-reaction[15] have not yet reached the level of precision where a firm value of n_* may be given. As we demonstrate later, the effect on the radiation spectrum is significant only for certain values of the cut-off.

3 Analytic estimate of the radiation spectrum

Analytic expressions for the spectrum of gravitational radiation emitted by a network of cosmic strings have been derived elsewhere[8,3]. While these analytic expressions are simplified for convenience, they offer the opportunity to examine the various dependencies of the spectrum on cosmic string and cosmological parameters.

The spectrum of gravitational radiation produced by a network of cosmic strings has two main features. First is the 'red noise' portion of the spectrum with nearly equal gravitational radiation energy density per logarithmic frequency interval, spanning the frequency range $10^{-8}\,\mathrm{Hz} \lesssim f \lesssim 10^{10}\,\mathrm{Hz}$. This spectrum corresponds to gravitational waves emitted during the radiation-dominated expansion era. This feature of the spectrum may be accessible to the forthcoming generation of gravitational wave detectors. Second is the peak in the spectrum near $f \sim 10^{-12}\,\mathrm{Hz}$. The amplitude and slope of the spectrum from the peak down to the flat portion of the spectrum is tightly constrained by the observed limits on pulsar timing noise.

3.1 Red noise portion of the spectrum

An analytic expression for the 'red noise' portion of the gravitational wave spectrum is given as follows:

$$\frac{f}{\rho_{\rm crit}}\frac{d\rho_{\rm gr}}{df} = \frac{8\pi}{9} A \frac{\Gamma(G\mu)^2}{\alpha c^4}\big[1 - \langle v^2\rangle/c^2\big]\frac{(\beta^{-3/2}-1)}{(z_{\rm eq}+1)} \qquad 10^{-8}\,\mathrm{Hz} \lesssim f \lesssim 10^{10}\,\mathrm{Hz} \tag{5}$$

$$A \equiv \rho_\infty d_H^2(t) c^2/\mu \qquad\qquad \beta \equiv \left[1 + f_{\rm r}\, \alpha d_H(t) c/(\Gamma G\mu t)\right]^{-1}. \tag{6}$$

In the above expressions, ρ_∞ is the energy density in 'infinite' or long cosmic strings, α is the invariant length of a loop as a fraction of the physical horizon length $d_H(t)$ at the time of formation, $\langle v^2\rangle$ is the rms velocity of the long strings, and $f_{\rm r} \approx 0.7$ is a correction for the damping of the relativistic center-of-mass velocity of newly formed string loops. All quantities are evaluated in the radiation era; $d_H(t) = 2ct$, $A = 52 \pm 10$, and $\langle v^2\rangle/c^2 = 0.43 \pm 0.02$ as obtained from numerical simulations[16].

The above expression for the spectrum has been obtained assuming no change in the number of relativistic degrees of freedom, g, of the background radiation-dominated fluid. However, the annihilation of massive particle species as the cosmological fluid cools leads to a decrease in the number of degrees of freedom, and a redshifting of all relativistic particles not thermally coupled to the fluid. This has the effect of modifying the amplitude of the spectral density[17], equation (5), by a factor $(g(T_f)/g(T_i))^{1/3}$ where $g(T_{i,f})$ is the number of degrees of freedom at temperatures before and after the annihilations. Using a minimal GUT particle physics model as the basis of the standard thermal history, we see that the red noise spectrum steps downwards with growing frequency.

$$\begin{aligned}
\frac{f}{\rho_{\rm crit}}\frac{d\rho_{\rm gr}}{df} \quad &= \quad \frac{8\pi}{9} A \frac{\Gamma(G\mu)^2}{\alpha c^4}\left[1 - \langle v^2\rangle/c^2\right]\frac{(\beta^{-3/2}-1)}{(z_{\rm eq}+1)} \\
&\times \begin{cases} 1 \\ \qquad 10^{-8}\,{\rm Hz} \lesssim f \lesssim 10^{-10}\alpha^{-1}\,{\rm Hz} \\ \\ (3.36/10.75)^{1/3} = 0.68 \\ \qquad 10^{-10}\alpha^{-1}\,{\rm Hz} \lesssim f \lesssim 10^{-4}\alpha^{-1}\,{\rm Hz} \\ \\ (3.36/106.75)^{1/3} = 0.32 \\ \qquad 10^{-4}\alpha^{-1}\,{\rm Hz} \lesssim f \lesssim 10^{8}\,{\rm Hz} \end{cases}
\end{aligned} \tag{7}$$

Hence, the red noise spectrum is sensitive to the thermal history of the cosmological fluid. The locations of the steps in the spectrum are determined by the number of relativistic degrees of freedom as a function of temperature, $g(T)$. As an example, we present the effect of a non-standard thermal history on the spectrum in Figure 1. In this sample model, the number of degrees of freedom $g(T)$ decreases by a factor of 10 at the temperatures $T = 10^9$, 10^5, $1\,{\rm GeV}$. The effect on the spectrum is a series of steps down in amplitude with increasing frequency; detection of such a shift would provide unique insight into the particle physics content of the early universe at temperatures much higher than

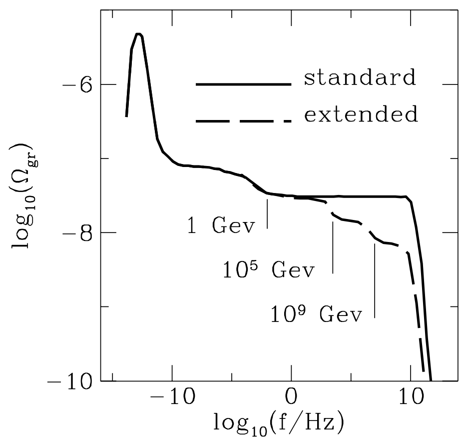

Figure 1: The effect of a non-standard thermal history of the cosmological fluid on the amplitude of the red noise portion of the gravitational wave spectrum is shown. The solid curve displays the spectrum produced using a minimal GUT with a maximum $g = 106.75$. The dashed curve shows the spectrum produced allowing for a hypothetical, non-standard evolution of $g(T)$, as might occur if there were a series of phase transitions, or a number of massive particle annihilations as the universe cooled. For temperatures $T > 10^9$ GeV, the number of degrees of freedom is $g = 10^4$. For 10^5 GeV $< T < 10^9$ GeV, $g = 10^3$. For $T < 10^5$GeV, the standard thermal scenario is resumed.

may be achieved by terrestrial particle accelerators. In the case of a cosmological model with a thermal history such that $g(T_i) \gg g(T_f)$ for $T_i > T_f$, all radiation emitted before the cosmological fluid cools to T_i will be redshifted away by the time the fluid reaches T_f. As we discuss later, such a sensitivity of the spectrum to the thermal history affects the nucleosynthesis bound on the total energy in gravitational radiation, and the opportunity to detect high frequency gravitational waves.

3.2 Peaked portion of the spectrum

We now turn our attention to the peaked portion of the gravitational wave spectrum. The shape of this portion depends critically on the model for the emission by a loop, presented in section 2. Specifically, if a loop emits in very high modes, a significant portion of the loop energy maybe radiated at high frequencies, well above the fundamental frequency of the loop. Hence, the

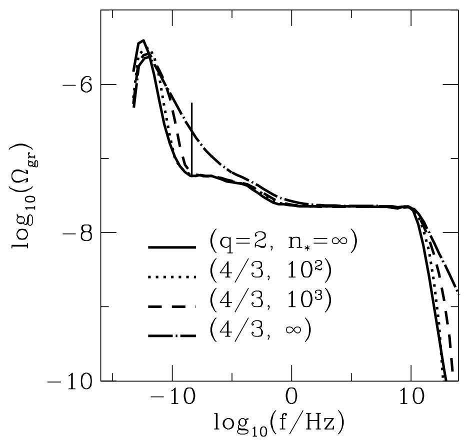

Figure 2: The effect of a cut-off in the radiation mode number on the spectrum of gravitational radiation is shown. Curves for the loop radiation spectral index $q = 2$, $4/3$ for various values of n_* are shown. The vertical line shows the location of the frequency bin probed by pulsar timing measurements. For $n_* \lesssim 10^2$ the shape of the spectrum is insensitive to the value of q for purposes of pulsar timing measurements. For increasing n_*, more radiation due to late-time cosmic string loops is emitted in the pulsar timing frequency band.

dominant behaviour of the peaked portion of the spectrum is given by

$$\frac{f}{\rho_{\rm crit}}\frac{d\rho_{\rm gr}}{df} \approx \begin{cases} C_1/f^{(q-1)} & 1 < q < 2 \\ C_2/f & q \geq 2 \end{cases}$$
$$\text{for} \quad 10^{-12}\,\text{Hz} \lesssim f \lesssim 10^{-8}\,\text{Hz}. \tag{8}$$

Here $C_{1,2}$ are dimensionful quantities which depend on $G\mu/c^2$, α, Γ, A, q and n_*. A lengthy expression displaying the full dependence of the spectrum on these parameters is not particularly enlightening, they can be found in ref.[13].The qualitative behaviour is as follows. The overall height of the spectrum depends linearly on $G\mu/c^2$, while the frequency at which the peaked spectrum gives way to the red noise spectrum depends inversely on α. The important result is that for values of the mode cut-off $n_* \lesssim 10^2$, the spectrum drops off as $1/f$ for any value $q \geq 4/3$. As a demonstration, sample spectra with various values of n_* are displayed in Figure 2. Hence, the introduction of a sufficiently low mode cut-off eliminates the dependence of the spectrum on q, the loop spectral index.

3.3 How stochastic is the background?

We now comment on the statistical properties of the background. We would like to know how good an approximation it is to claim that the signal is stochastic. The relevant quantity to consider is the number of string loops which contribute to the radiation produced in a particular frequency bin. If we identify, for each frequency f observed, a unique time of emission corresponding to the time $t_{\rm f}$ that the loop formed, then one can calculate the number of individual horizon cells on the sky contributing to the radiation background at each frequency:

$$N(f) \approx 4 \times 10^{17} \alpha^2 \left(\frac{f}{f_{\rm p}} \right)^2 . \tag{9}$$

Here, $f_{\rm p} \approx 4 \times 10^{-9}$Hz is the frequency corresponding to horizon sized gravitational waves that would be detected by the milli-second pulsar and α is the loop production size with respect to the horizon. For the frequencies of interest, $f \approx 100$Hz (LIGO/VIRGO), $f \approx 10^{-3}$Hz (LISA) and $f \approx 4 \times 10^{-9}$Hz (pulsar), the values of N are 10^{28}, 10^{18} and 4×10^{7} respectively, where we have used $\alpha \sim 10^{-5}$ for illustrative purposes. Hence, we may argue from the central limit theorem, that if the N cells act independently, the sum of the amplitudes of radiation arriving at the detector or pulsar is gaussian and the background is stochastic. This gaussian assumption may break down at a frequency of 10^{-12}Hz where $N \sim 1$, corresponding to radiation emitted around a redshift $z \sim 4$. In this case, the radiation is due to a relatively small number of sources which are on average a few hundred megaparsecs away. These sources would appear as burst sources at a detector, rather than continuous noise. Unfortunately, this region of the spectrum is outside the range of interferometers since the timescales ($\sim$ 10,000 years) required for a detection are too large, but it may be possible to detect these gravitational waves using, for example, gravitational lensing.

4 Alternative scenarios

4.1 Open universes

There is a substantial body of astronomical evidence which suggests that the cosmological density parameter is less than critical, $\Omega < 1$. Therefore, it seems sensible to consider the effect of a low-density universe on the spectrum of gravitational radiation.

The evolution of strings in an open universe will be very much the same as for the flat case, except that after curvature domination at $t_{\rm curv} \sim \Omega_0 t_0$,

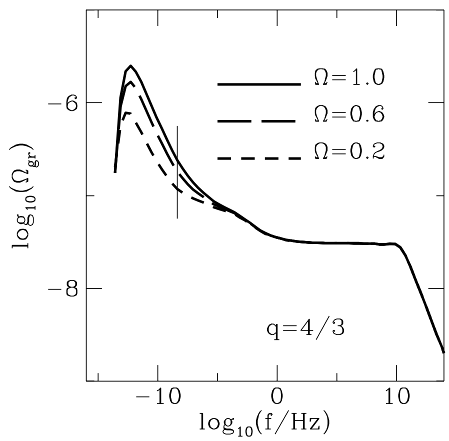

Figure 3: The effect of a low density, $\Omega_0 < 1$ universe on the peaked portion of the gravitational wave spectrum. The solid, long- and short-dashed curves represent spectra for $\Omega_0 = 1$, 0.6, 0.2. The vertical line shows the location of the frequency bin probed by pulsar timing measurements. For the loop spectral index $q = 2$, a low density universe dilutes only the lowest frequency waves, corresponding the radiation emitted by loops still present today.

where $a(t) \propto t$, the linear scaling regime no longer exists. In fact, it has been shown[18] that the characteristic length-scale of the network will increase like $L \sim t[\log(t)]^{1/2}$, rather than $L \sim t$, and hence the long string density $\rho_\infty \propto L^{-2} \propto [t^2 \log(t)]^{-1}$, decreases relative the critical density $\rho_{\rm crit} \propto t^{-2}$, but increases with respect to the background density $\rho_0 \propto t^{-3}$. This is important for consideration of the gravitational waves created with low frequencies in the matter era and also for the normalization of the cosmic microwave background[19] as discussed in section 5. However, from the point of view of the red noise spectrum, which is produced in the radiation dominated era, there is little difference apart from a shift in the frequency corresponding to equal matter-radiation.

Using methods similar to those used in the previous section, but modified to accommodate $\Omega_0 < 1$, we have examined the spectrum of gravitational radiation produced by the cosmic string network in an open FRW space-time, with $0.1 < \Omega_0 < 1$. For this range of values of the cosmological density parameter, the portion of the spectrum produced at a time t is shifted downward by a factor $\sim \Omega(t)$. Sample spectra for various values of Ω_0 are displayed in Figures

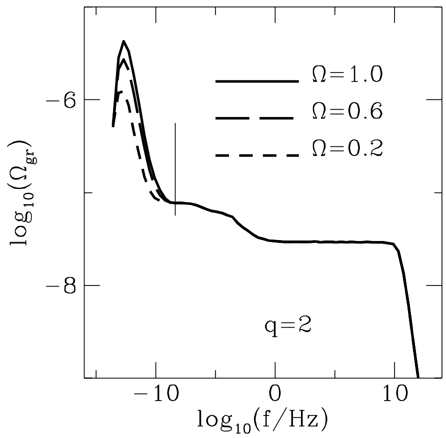

Figure 4: The effect of a low density, $\Omega_0 < 1$ universe on the peaked portion of the gravitational wave spectrum. The solid, long- and short-dashed curves represent spectra for $\Omega_0 = 1$, 0.6, 0.2. The vertical line shows the location of the frequency bin probed by pulsar timing measurements. For the loop spectral index $q = 4/3$, a low density universe leads to a dilution of gravitational waves with wavelengths up to $f \sim 10^{-5}$ Hz.

3-4. In the case that the spectrum drops off slower than $1/f$, that is $1 < q < 2$ and $n_* \to \infty$, the spectral density at frequencies as high as $f \sim 10^{-5}$ Hz is diluted for $\Omega_0 < 1$. In the case that the spectrum drops off as $1/f$, that is $q > 2$, only at lower frequencies, $f \lesssim 10^{-10}$ Hz, is the spectral density affected. This behaviour is exactly as expected: only those frequencies dominated by matter eras loops are affected.

4.2 Hybrid defects

Cosmic strings are generic in a number of symmetry breaking schemes where the first homotopy group is non-trivial. Simple examples where this can take place are, when the original symmetry group contains a $U(1)$ subgroup which is broken to the identity or when the broken symmetry group has a Z_2 subgroup. However, if these simple examples occur as part of a much more complicated scheme, it is likely that hybrid systems such as strings connected to either domain walls or monopoles may form. In the case when the hybrid system does not annihilate immediately this may lead to a stochastic background in

a similar way to that for strings[20]. We will, therefore, illustrate the basic physics of this possibility by reference to the two examples already discussed. The interesting feature of these models being that they evade the constraints from the cosmic microwave background and pulsar timing allowing for larger values of $G\mu/c^2$ and hence larger contributions to the stochastic background in the detectable range of frequencies.

The first and simplest case is that of domain walls connected to strings. This will occur in the following symmetry breaking scheme,

$$G \rightarrow H \times Z_2 \rightarrow H \,, \tag{10}$$

where at the first transition ($T = \eta_{\rm s}$) strings form and at the second ($T = \eta_{\rm w}$) each string gets attached to a domain wall. Assuming that the domain walls form during the radiation era, then before the formation of walls the strings will evolve as in the standard scenario, creating a nearly flat stochastic background in the frequency range $f \in [2/\alpha(t_{\rm w}t_{\rm eq})^{1/2}, 2/\alpha(t_* t_{\rm eq})^{1/2}]$, where $t_{\rm d} = \min[\mu/\sigma, t_{\rm w}]$ is the time at which the walls dominate the dynamics of the network, $t_{\rm w}$ is the time of wall formation, t_* is the time when relativistic evolution of the string network begins and $\mu \sim \eta_{\rm s}^2$, $\sigma \sim \eta_{\rm w}^3$ are the string tension and wall surface tension respectively. Once the walls dominate the dynamics of the network, it will break up into what are effectively string loops spanned by domain walls. These string loops will collapse into gravitational radiation and other decay products, such as gauge particles, in about a Hubble time. The exact nature of this contribution is obviously dependent on the distribution of the loops produced by the fragmentation process. This process takes place over a relatively short frequency range and may lead to a sharp spike in the spectrum. If a sharp peak in the background spectrum was detected, the the demise of hybrid defects might provide one potential explanation. However, unlike the flat string background, the sensitive dependence on model phenomenology in this case, means that it is not possible at present to predict whether or where such a peak should occur.

The second possibility is of strings connected by monopoles which would have been created in a symmetry breaking scheme such as,

$$G \rightarrow H \times U(1) \rightarrow H \,, \tag{11}$$

where monopoles are formed at the first transition ($T = \eta_{\rm m}$) and strings connected between monopole/anti-monopole pairs are formed at the second ($T = \eta_{\rm s}$). In this case there is no period in which the strings behave as they do in the standard scenario. If the first transition is after any period of inflation which may have taken place, then the average separation of the monopoles

is less the Hubble radius and the hybrid system will collapse in one Hubble time by dissipating energy into friction with the cosmological fluid. However, if the monopoles are formed during inflation then the system does not collapse immediately, allowing the formation of a stochastic background. This scenario has been studied in detail[21] for the simplest case of a straight string connecting two monopoles. It was found that the power spectrum emitted by this simple configuration is divergent with $P_n \propto n^{-1}$ due to the perfect symmetry (a similar spectrum is emitted by a perfectly circular string loop). Assuming that this divergence is softened in a less symmetric case, one may be able to calculate accurately the stochastic background created. However, it is likely to be dependent on the initial distribution of monopoles created in the first phase transition.

5 Observational bounds on the radiation spectrum

In this section we determine the observational constraint on the cosmic string mass-per-unit-length $G\mu/c^2$. To begin, we discuss the recent analyses of the pulsar timing data, after which we apply the newly obtained bounds to the cosmic string gravitational wave background.

The observations used to place a limit on the amplitude of a stochastic gravitational wave background[5] consist of pulse arrival times for PSR B1937+21 and PSR B1855+09. Although there has been some recent controversy regarding the analysis of the pulsar timing data[6], the work by McHugh *et al*[4] best assesses the likelihood that the timing residuals are due to gravitational radiation. We note that all analyses to date have assumed a flat, red noise spectrum for the gravitational wave spectral density. Such an assumption is only justified for a restricted range of frequencies in the case of a background due to cosmic strings, as we have demonstrated in the preceding section. Hence, a statistical analysis which uses a realistic model of the cosmic string spectrum may obtain a different limit on the amplitude of the spectral density.

We now present values of the parameter $G\mu/c^2$ for values of α which satisfy the pulsar timing constraint on the gravitational radiation spectrum. Contours of constant $\Omega_{\rm gr}$ in the logarithmic frequency bin $f = (8\,{\rm yrs})^{-1}$, given by (1), in $(\alpha, G\mu/c^2)$ parameter space, are shown in Figure 5. We have used cosmological parameters $\Omega_0 = 1$ and $h \in [0.5, 0.75]$ with the cosmic string loop radiation efficiency $\Gamma = 60$. We find

$$G\mu/c^2 < \begin{cases} 2.0(\pm 0.4)\times 10^{-6}\ (2h)^{-8/3} & q = 4/3 \\ 5.4(\pm 1.1)\times 10^{-6} & q \geq 2 \text{ or } n_* \lesssim 10^2. \end{cases} \tag{12}$$

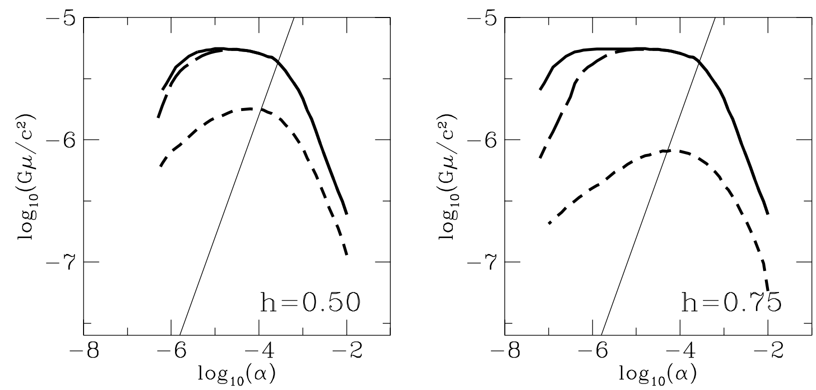

Figure 5: Curves of constant $\Omega_{\rm gr}$ in $(\alpha, G\mu/c^2)$ parameter space are shown. For a given value of α, these figures give the observational bound on $G\mu/c^2$ in the case $h = 0.5$, 0.75. In each figure, the constraining curves for $q = 10$, 2, 4/3 are given by the solid, long-, and short-dashed curves. The light dashed lines show $\alpha = \Gamma G\mu/c^2$. The most conservative constraint is $G\mu/c^2 < 5.4 \times 10^{-6}$.

These constraints correspond to the maximum value of $G\mu/c^2$ along the contour of constant $\Omega_{\rm gr}$. In the case $q = 4/3$, this maximum occurs near $\alpha = \Gamma G\mu/c^2$, the expected size of newly formed loops based on considerations of the gravitational back-reaction, while for the $q \geq 2$ or $n_* \lesssim 10^2$ case, the maximum occurs at a slightly smaller value of α. For both larger and smaller values of α the bounds become more stringent[8]. The unusual dependence on h is due to the contribution from high mode number waves emitted in the matter era, for which the amplitude depends on both the slope of the spectrum and the time of radiation-matter equality. The quoted errors are due to uncertainties in the cosmic string model parameters measured by the numerical simulations[16]. For the case of an open universe, there is no change in the $q \geq 2$ or $n_* \lesssim 10^2$ bound. However, the $q = 4/3$ bound is weakened by a factor $\sim 1/\Omega_0$.

We now comment on the validity of the model which we have used to generate the gravitational radiation spectra. We have shown that the observational bounds on the total spectrum are sensitive to the value of the loop spectral index q, unless there is a back-reaction cut-off $n_* \lesssim 10^2$. Furthermore, we have noted that there is uncertainty in the characteristic value of the loop spectral index, q. Hence, we feel that it is more reasonable to take the conservative bound of (12) at the present. Next, consider the extended one-scale

model[3,8], for the evolution of the string network. This model assumes that the long string energy density scales relative to the background energy density, with the dominant energy loss mechanism due to the formation of loops of a characteristic scale. A more sophisticated model, by Austin *et al* [22], attempts to include the effect of the gravitational back-reaction on the long-term evolution of the string network; results suggest that an effect of the back-reaction may be to lower the scaling density in long strings at late times, beyond the reach of numerical simulations. Hence, there is some uncertainty as to how accurately the extended one-scale model describes the evolution of the string network. However, we do not believe that these considerations could result in a decrease in the amplitude of the gravitational wave background by more than $\sim 50\%$. Thus, we quote $G\mu/c^2 < 5.4(\pm 1.1)\times 10^{-6}$ as a conservative bound on the cosmic string mass-per-unit-length.

The bounds already computed[8] due to the constraint on the total energy density in gravitational waves at the time of nucleosynthesis remain valid. For a limit on the effective number of neutrino species $N_\nu < 3.1, 3.3, 3.6$, the bound on the cosmic string mass-per-unit-length is $G\mu/c^2 < 2, 6, 10\times 10^{-6}$ respectively, evaluated at $\alpha = \Gamma G\mu/c^2$. The big-bang nucleosynthesis limit on the number of effective neutrino species is a conservative $N_\nu < 4$, owing to uncertainties in the systematic errors in the observations of light element abundances[23]. Hence, until the observations are refined, the nucleosynthesis bound is weaker than the pulsar timing bound. Furthermore, the translation of the limit on N_ν into the bound on $G\mu/c^2$ is sensitive to the thermal history of the cosmological fluid[17]. The bound on the string mass-per-unit-length may be considerably weakened if the cosmological fluid possessed many more relativistic degrees of freedom in the early universe beyond those given by a minimal GUT model.

Comparing detailed computations of the large angular scale cosmic microwave background temperature anisotropies induced by cosmic strings[24] with observations, the cosmic string mass-per-unit-length has been normalized to

$$G\mu/c^2 = 1.05\,^{+0.35}_{-0.20}\times 10^{-6}. \tag{13}$$

In an open universe, this normalization is expected to increase slightly[19] to $G\mu/c^2 \sim 1.6\times 10^{-6}$ for $\Omega_o = 0.1$. Therefore, given the uncertainties in the extended one-scale model, we find the gravitational radiation spectrum to be compatible with observations.

We have already noted that the hybrid systems evade the constraint from pulsar timing unless they survive until around the time of nucleosynthesis ($t_{\rm nucl} \sim 1$sec), much later than thought possible according to the standard model of particle physics. Therefore, the only constraint on the stochastic

background comes from Big Bang nucleosynthesis. The interesting thing to note is that since the hybrid network will annihilate at some time $t_{\rm a}$, well before nucleosynthesis, then assuming that the spectrum is flat, the amplitude of the stochastic background allowed is larger by a factor of

$$\log\left(\frac{t_{\rm nucl}}{t_*}\right) \Big/ \log\left(\frac{t_{\rm a}}{t_*}\right) . \tag{14}$$

For the first transition at the GUT scale, that is $t_* \sim 10^{-32}$, and the second around 10^{10}GeV with almost immediate annihilation, then this results in the constraint being weakened by approximately a factor of 10, corresponding to $G\mu \sim 10^{-5}$. This makes a detection in the first generation of LIGO detectors marginally possible[20], although the parameter values would have to be rather extreme.

6 Detection of the Radiation Spectrum

We would like to determine whether the stochastic gravitational wave spectrum emitted by cosmic strings may be observed by current and planned detectors. Because all ground-based detectors operate at frequencies $f \gtrsim 10^{-3}$ Hz, we need only consider the 'red noise' portion of the gravitational wave spectrum (7). Noting that the spectral density, $\Omega_{\rm gr}(f)$, has a minimum value when $\alpha \to 0$ the predicted spectrum is bounded from below by

$$\begin{aligned} \Omega_{\rm gr}(f) &\geq \frac{24\pi}{9} A f_{\rm r} \frac{(1-\langle v^2\rangle/c^2)}{(1+z_{\rm eq})} (G\mu/c^2) \Big(g(T_0)/g(T_{\rm GUT})\Big)^{1/3} \\ &\geq 1.4\times 10^{-9} \qquad \text{for} \quad 10^{-8}\,\text{Hz} \lesssim f \lesssim 10^{10}\,\text{Hz}. \end{aligned} \tag{15}$$

Here we have used the normalization in (13) for $G\mu/c^2$, Hubble parameter $h = 0.75$, and assumed a minimal GUT thermal history. Hence, this lower bound is valid up to frequencies $f \sim 10^{-3}\alpha^{-1}\,$Hz $\sim 10\,$Hz based on our knowledge of the number of relativistic degrees of freedom, g, of the primordial fluid up to temperatures $T \sim 10^3\,$GeV. Notice that a measurement of the spectral density due to cosmic strings at higher frequencies would sample g at higher temperatures.

We may in turn place a lower bound on the amplitude of the dimensionless strain predicted for the gravitational wave emitted by cosmic strings:

$$h_{\rm c} = 1.3\times 10^{-18} h \sqrt{\Omega_{\rm gr}(f)} \Big(\frac{f}{1\,\text{Hz}}\Big)^{-1}$$

$$\geq \quad 3.6 \times 10^{-23} \Big(\frac{f}{1\,\mathrm{Hz}}\Big)^{-1} \qquad \text{for} \quad 10^{-6}\,\mathrm{Hz} \lesssim f \lesssim 10^{8}\,\mathrm{Hz}. \tag{16}$$

The expressions (15-16) are useful for comparison with the planned sensitivities of the forthcoming generation of gravitational wave detectors[25].

The most promising opportunity to probe for a stochastic gravitational wave background due to cosmic strings is through a cross-correlation of the observations of the advanced LIGO, VIRGO and LISA interferometers. It is estimated that the advanced LIGO detectors will have the sensitivity

$$h_{3/\mathrm{yr}} = 5.2 \times 10^{-25} \Big(\frac{f}{1\,\mathrm{kHz}}\Big)^{1/2} \tag{17}$$

for stochastic waves (equation 125c of Thorne[26]), sufficient to measure the minimum predicted strain (16) near $f \sim 100\,\mathrm{Hz}$ in a 1/3-year integration time. More recent calculations[27] confirm that the orientation of the advanced LIGO interferometers will be sufficient in order to detect the cosmic string gravitational wave background. For the LISA project[28], comparison of the projected strain sensitivity $h_c \sim 10^{-20}$ at the frequency $f \sim 10^{-3}\,\mathrm{Hz}$ with (16) indicates that the space-based interferometer will be capable of detecting a gravitational radiation background produced by a network of cosmic strings.

Other ground-based interferometric gravitational wave detectors are in development or under construction. The GEO600 and TAMA300 detectors, operating near frequencies $f \sim 10^3\,\mathrm{Hz}$, may also be capable of measuring a cosmic string generated background.

A network of resonant mass antennae, such as bar and TIGA detectors may probe for a stochastic background. Successful detection by these antennae will require improved sensitivity and longer integration time. However, cross-correlation between a narrow-band bar and a wide-band interferometric detector may improve the opportunities. Estimates of the sensitivity of such a system, assuming optimum detector alignment[29], indicate that

$$\sqrt{h_{\mathrm{int}} h_{\mathrm{b}}} \gtrsim 2.6 \times 10^{-19} \sqrt{\Omega_{\mathrm{gr}}(f)} \Big(\frac{f}{1\,\mathrm{kHz}}\Big)^{-3/2} \Big(\frac{t_{\mathrm{obs}}}{10^7\,\mathrm{s}}\Big)^{1/2} \tag{18}$$

is necessary to detect a background Ω_{gr}. Hence, for a 1/3-year observation time, the bar and interferometer strain sensitivities at 1 kHz must be better than $\sim 10^{-23}$ in order to detect the cosmic string background.

We stress that the amplitude of the cosmic string gravitational wave background for frequencies $f \gtrsim 10\,\mathrm{Hz}$ is sensitive to the number of degrees of freedom of the cosmological fluid at temperatures $T \gtrsim 10^3\,\mathrm{GeV}$. The amplitude of the cosmic string background at LISA-frequencies, near $f \sim 10^{-3}\,\mathrm{Hz}$, is firm, since

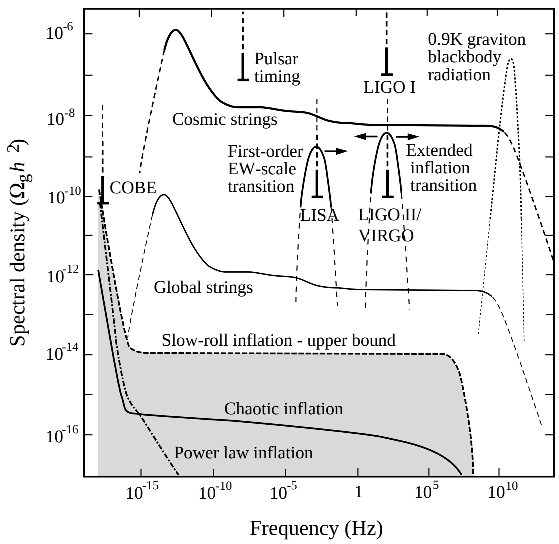

Figure 6: Summary of the potential cosmological sources of a stochastic gravitational radiation background, including inflationary models, first order phase transitions and cosmic strings, as well as a primordial 0.9K blackbody graviton spectrum (the analogue of the blackbody photon radiation). Also plotted are the relevant constraints from the COBE measurements, pulsar timings, and the sensitivities of the proposed interferometers. Notice that local cosmic strings and strongly first-order phase transitions may produce detectable backgrounds in contrast to standard slow-roll inflation models.

the cosmological fluid near the temperature $T \sim 10\,\mathrm{MeV}$ is well understood. However, at the higher frequencies probed by ground-based detectors, our uncertainty in the number of degrees of freedom of the cosmological fluid, as determined by the correct model of particle physics at that energy scale, may reduce the predicted amplitude (15) of gravitational radiation.

7 Conclusion

We have presented improved calculations of the spectrum of relic gravitational waves emitted by cosmic strings. We demonstrated that the effect of a gravitational back-reaction on the radiation spectrum of cosmic string loops, for which there is an effective mode cut-off $n_* \lesssim 10^2$, may serve to weaken the

pulsar timing bound on the cosmic string mass-per-unit-length. Arguing for a model of radiation by loops, for which either the spectral index is $q \geq 2$ or there is an emission mode cut-off $n_* \lesssim 10^2$, we obtain the conservative bound $G\mu/c^2 < 5.4(\pm 1.1) \times 10^{-6}$ due to observations of pulsar timing residuals. We believe this bound to be robust, in that the spectrum depends weakly on the precise value of the mode cut-off, up to $n_* \sim 10^2$. We have noted the interesting result that the flat, red noise portion of the gravitational wave spectrum is sensitive to the thermal history of the cosmological fluid, revealing features of the particle physics content at early times. We have also discussed the possibility of a low-Ω universe and hybrid systems of defects, such as strings connected by domain walls or monopoles. Finally, we have pointed out that the generation of advanced LIGO, VIRGO and LISA interferometers should be capable of detecting the predicted stochastic gravitational wave background due to cosmic strings.

We may place the cosmic string scenario in context with other candidate sources for a stochastic background of cosmological gravitational waves[30] in Fig. 6. These include gravitational waves created during inflation[31] and through bubble collisions following a first-order phase transition[32]. Viable theoretical scenarios within detector sensitivity include strongly first-order phase transitions, possibly at the end of inflation, and networks of cosmic strings. At this stage, other primordial backgrounds from slow-roll inflation, global topological defects and the standard electroweak phase transition appear to be out of range. The discovery of any of the possible cosmological sources will have enormous implications for our understanding of the very early universe and for fundamental physics at the highest energies.

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